Sector-level Attribution Effects with Compounded Notional Portfolios

Why Would We Want Them
and
How Can We Get Them?

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The Setup – What is Arithmetic Time Period Linking Trying to Accomplish?

- Additivity
  - of sectors to the total portfolio
  - of attribution effects to the total value add
  - of time periods to the total attribution period
- As contrasted to geometric attribution methods…

Single Period Sector Performance…

Is easy. For Portfolio P:

Period t	**Return**	**Weight**	**Contribution**
Sector i			
Sector i	$R_{P,i,t}$	$W_{P,i,t}$	$C_{P,i,t} = W_{P,i,t} * R_{P,i,t}$
Sector i			
Total			$R_{P,t} = \sum_i C_{P,i,t}$

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Multi-Period Sector Performance…

Is easy. For Portfolio P:

	Period 1				Period t				Full Performance Period 0 - t
	R	W	C	Adjusted Contribution	R	W	C	Adjusted Contribution	Adjusted Contribution
Sector i				$\tilde{C}_{P,i,t} = C_{P,i,t} * (1 + \overline{R}_{P,t-1})$				$\tilde{C}_{P,i,t} = C_{P,i,t} * (1 + \overline{R}_{P,t-1})$	$\tilde{C}_{P,i} = \sum_t \tilde{C}_{P,i,t}$
Sector i									
Sector i									
TOTAL				$\tilde{C}_{P,t} = \sum_i \tilde{C}_{P,i,t}$				$\tilde{C}_{P,t} = \sum_i \tilde{C}_{P,i,t}$	$R_P = \sum_i \tilde{C}_{P,i} = \sum_t \tilde{C}_{P,t}$

Multi-Period Sector Performance - 2

- "Adjusted" contributions are scaled to prior cumulative Portfolio return:

$$\overline{R}_{P,t} = \left[\prod_{s=1}^{t} (1 + R_{P,s}) \right] - 1$$

- Consistent with intuition for dollar contributions, which <u>are</u> additive: 10% return on \$100 = \$10 in period 1 makes 10% return in period 2 "worth" \$11, or 11% in base-period terms.

Single Period Sector Attribution…

Is easy.

Period t	Portfolio P			Benchmark B			Attribution Effects			Value Added
	R	W	C	R	W	C	Allocation	Selection	Interaction	
Sector i							$A_{i,t}$	$S_{i,t}$	$I_{i,t}$	$V_{i,t} = C_{P,i,t} - C_{B,i,t}$ $= A_{i,t} + S_{i,t} + I_{i,t}$
Sector i										
Sector i										
Total							$A_t = \sum_i A_{i,t}$	$S_t = \sum_i S_{i,t}$	$I_t = \sum_i I_{i,t}$	$V_t = \sum_i V_{i,t}$ $= A_t + S_t + I_t$

Single Period Attribution - 2

- Using the familiar, “vanilla” Brinson method:

$$A_{i,t} = (W_{P,i,t} - W_{B,i,t}) * R_{B,i,t}$$
$$S_{i,t} = W_{B,i,t} * (R_{P,i,t} - R_{B,i,t})$$
$$I_{i,t} = (W_{P,i,t} - W_{B,i,t}) * (R_{P,i,t} - R_{B,i,t})$$

- Many use Brinson-Fachler, in which:

$$A_{i,t} = (W_{P,i,t} - W_{B,i,t}) * (R_{B,i,t} - R_{B,t})$$

- but then

$$V_{i,t} = C_{P,i,t} - C_{B,i,t} \neq A_{i,t} + S_{i,t} + I_{i,t}$$

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Multi-Period Sector Attribution

Is hard!

	Period 1				Period t				Full Period Attribution 0 - t			
	A	S	I	V	A	S	I	V	Allocation	Selection	Interaction	Value Added
Sector i									$\tilde{A}_i = ???$	$\tilde{S}_i = ???$	$\tilde{I}_i = ???$	$V_i = \tilde{C}_{P,i} - \tilde{C}_{B,i} = \tilde{A}_i + \tilde{S}_i + \tilde{I}_i$
Sector i												
Sector i												
Total									$A = \sum_i \tilde{A}_i$	$S = \sum_i \tilde{S}_i$	$I = \sum_i \tilde{I}_i$	$V = R_P - R_B = A + S + I$

Multi-period Sector Attribution - 2

- It's hard, because the standard Brinson formulas include weight & return from <u>two</u> entities, the Portfolio and the Benchmark
- What is the "adjustment" factor when these two entities do not track?

Solutions: A Simple Attempt

- Just use the prior cumulative Portfolio return, like we did with single period Portfolio performance:

$$\tilde{A}_{i,t} = A_{i,t} * (1 + \overline{R}_{P,t-1})$$
$$\tilde{S}_{i,t} = S_{i,t} * (1 + \overline{R}_{P,t-1})$$
$$\tilde{I}_{i,t} = I_{i,t} * (1 + \overline{R}_{P,t-1})$$

- Not exact
- The further Portfolio and Benchmark returns drift, the worse it gets.

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Something a Tad More Sophisticated?

- Scale the weights by their respective entity's prior cumulative performance:

$$\tilde{A}_{i,t} = [(W_{P,i,t} * (1 + \overline{R}_{P,t-1})) - (W_{B,i,t}(1 + \overline{R}_{B,t-1}))] * R_{B,i,t}$$

$$\tilde{S}_{i,t} = [(W_{B,i,t}(1 + \overline{R}_{B,t-1}))] * (R_{P,i,t} - R_{B,i,t})$$

$$\tilde{I}_{i,t} = [(W_{P,i,t} * (1 + \overline{R}_{P,t-1})) - (W_{B,i,t}(1 + \overline{R}_{B,t-1}))] * (R_{P,i,t} - R_{B,i,t})$$

- Still not exact
- There is an algebraic solution for the error, but it is hard to explain, and can be larger than the effect itself.

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The First Real Deal: Cariño

- Cariño, David, "Combining Attribution Effects over Time", *The Journal of Performance Measurement,* Summer 1999
- Attempts to solve by viewing continuously compounding effects

$$\{\tilde{A}_{i,t}, \tilde{S}_{i,t}, \tilde{I}_{i,t}\} = \left[\frac{\ln(1+R_{P,t}) - \ln(1+R_{B,t})}{R_{P,t} - R_{B,t}}\right] * \{A_{i,t}, S_{i,t}, I_{i,t}\}$$

- But the approach still leaves an "unexplained residual … it is fair to distribute the residual proportionately".
- Hence, a final re-adjustment occurs after summing up the adjusted effects:

$$\{\tilde{A}_{i}, \tilde{S}_{i}, \tilde{I}_{i}\} = \sum_t \left[\{\tilde{A}_{i,t}, \tilde{S}_{i,t}, \tilde{I}_{i,t}\}\right] / \left[\frac{\ln(1+R_P) - \ln(1+R_B)}{R_P - R_B}\right]$$

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Menchero

- Menchero, Jose, "An Optimized Approach to Linking Attribution Effects over Time," *The Journal of Performance Measurement*, Fall 2000
- Based on geometric compounding, constructs a scaling factor, such that:

$$\{\tilde{A}_{i,t}, \tilde{S}_{i,t}, \tilde{I}_{i,t}\} = F * \{A_{i,t}, S_{i,t}, I_{i,t}\} \qquad F = \frac{1}{T}\left[\frac{R_P - R_B}{(1+R_P)^{1/T} - (1+R_B)^{1/T}}\right]$$

- But again, "still leaves a small residual … introduce a set of corrective terms α_t that distribute this small residual among the different periods so that the following equation exactly holds"

$$R_P - R_B = \sum_t (F * \alpha_t) * (R_{P,t} - R_{B,t})$$

- And proceeds by optimizing the residual to make α_t as small as possible

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Frongello, Wilshire

- Frongello, Andrew, "Linking Single Period Attribution Results," *The Journal of Performance Measurement*, Spring 2002
- Bonafede, Julia K., Steven J. Foresti, and Peter Matheos, "A Multi-period Linking Algorithm That Has Stood the Test of Time," *The Journal of Performance Measurement*, Fall 2002

$$\{\tilde{A}_{i,t}, \tilde{S}_{i,t}, \tilde{I}_{i,t}\} = \{A_{i,t}, S_{i,t}, I_{i,t}\} * \left[\prod_{j=1}^{t-1} 1 + R_{P,t}\right] + R_{B,t} * \left[\sum_{j=1}^{t-1} \{\tilde{A}_{i,t}, \tilde{S}_{i,t}, \tilde{I}_{i,t}\}\right]$$

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Frongello, Wilshire - 2

<table>
<tr><td colspan="2" rowspan="2">Sources of this period value added</td><td colspan="4">This period portfolio return =</td></tr>
<tr><td>This period Benchmark</td><td>This period Allocation</td><td>This period Selection</td><td>This period Interaction</td></tr>
<tr><td rowspan="4">Cumulative Prior Portfolio Return =</td><td>Cumulative Benchmark</td><td>Benchmark</td><td rowspan="4">Allocation</td><td rowspan="4">Selection</td><td rowspan="4">Interaction</td></tr>
<tr><td>Cumulative Allocation</td><td>Allocation</td></tr>
<tr><td>Cumulative Selection</td><td>Selection</td></tr>
<tr><td>Cumulative Interaction</td><td>Interaction</td></tr>
</table>

- Decomposes a periods attribution effect into:
  - This period’s effect * cumulative prior portfolio return
  - Plus cumulative prior periods’ effect * this period’s benchmark return
- Valtonnen later shows that this is a valid though arbitrary decomposition, and is one of a continuum of exact solutions

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Davies & Laker

- Davies, Owen and Laker, Damien, “Multiple-period Performance Attribution Using the Brinson Model”, *The Journal of Performance Measurement*, Fall 2001
- Goes back to the “first principles” of Brinson, Hood, Beebower (1986), defining “Notional Portfolios”:

	Returns of Portfolio	Returns of Benchmark
Weights of Portfolio	**Portfolio**	**Notional Allocation**
Weights of Benchmark	**Notional Selection**	**Benchmark**

- In period t, then,

$$A_t = R_{A,t} - R_{B,t}$$

$$S_t = R_{S,t} - R_{B,t}$$

$$I_t = R_{P,t} + R_{B,t} - R_{A,t} - R_{S,t}$$

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Compounded Notional Portfolios

- Davies & Laker called it the "Exact Brinson Method"
- Currently referred to by this more neutral moniker
- Stated that any linking methodology, however it works, should equal the results of CNP, or it isn't Brinson
- Has intuitive appeal based on its real-world feasibility

CNP Doesn’t Do Sector-Level?

- But, as late as Summer of 2005, the primary downside of CNP was that no one had put forth a method of producing sector-level attribution effects that summed to the total portfolio effects.
  - Actually, Laker himself showed an example using Cariño under CNP, but it wasn’t exact
  - Valtonnen showed Frongello under CNP. Exact, but still a hybrid – and the interaction effect was a monster.

The Solution

- You've probably seen, however, that we already solved this problem back on page 4
- Since with CNP we are dealing with four <u>individual</u> portfolios (even if two of them are notional), we can simply apply the multi-period single portfolio method to each of them, and apply the "first principles" Brinson:

$$\tilde{A}_{i,t} = \tilde{C}_{A,i,t} - \tilde{C}_{B,i,t}$$

$$\tilde{S}_{i,t} = \tilde{C}_{S,i,t} - \tilde{C}_{B,i,t}$$

$$\tilde{I}_{i,t} = \tilde{C}_{P,i,t} + \tilde{C}_{B,i,t} - \tilde{C}_{A,i,t} - \tilde{C}_{S,i,t}$$

- And everything sums exactly every which way

CNP vs. Other Methods

- Robustness, Absence of Residuals:
  - Equivalent
- Intuitiveness:
  - Superior, IMHO
- Transparency:
  - Superior, by virtue of simplicity
- Commutativity:
  - "simply interchanging two of the periods should not change the results".
  - Only Frongello is not commutative, and he argues that that is a desirable aspect, calling it "Order Dependence"

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CNP vs. Other Methods - 2

- Metric Preservation
  - "Two periods that have identical relative performance should contribute equally to relative performance when they are linked together."
  - This criteria, advanced by Menchero, is only evidenced in Menchero's method
- A-causality
  - "August's stock selection contribution to this year's excess return does not become available until after the end of December"
  - Put another way, a report produced at the end of May will have different numbers for May's attribution effects than a report produced at the end of June
  - IMHO, a big deal
  - Cariño and Menchero both exhibit a-causality

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Biggest Remaining Issue with CNP:

- **<u>Spurious Interaction Effects</u>**
  - Interaction appears over multiple periods, even when no single period exhibits Interaction at the Total Portfolio level.
  - Laker later addresses persuasively, by pointing out that Interaction arises not only from <u>simultaneous</u> effect of Allocation and Selection, but also from combined effects <u>over multiple periods</u>.
  - Frongello has interesting example, where Interaction effects in separate sectors exactly cancel each other out. Can produce alarmingly large Interaction effects over multiple periods.

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